Pre-Calculus Graded Course of Study

Conceptual Category: **Number and Quantity**	
Domain: The Complex Number System	**N-CN**
Clusters	• Perform arithmetic operations with complex numbers.
	• Represent complex numbers and their operations on the complex plane.

Conceptual Category: **Number and Quantity**	
Domain: Vector Quantities and Matrices	**N-VM**
Clusters	• Represent and model with vector quantities.
	• Perform operations on vectors.
	• Perform operations on matrices and use matrices in applications.

Conceptual Category: **Algebra**	
Domain: Seeing Structure in Expressions	**A-SSE**
Clusters	• Interpret the structure of expressions

Conceptual Category: **Algebra**	
Domain: Arithmetic with Polynomials and Rational Expressions	**A-APR**
Clusters	• Rewrite rational expressions

Conceptual Category: **Algebra**	
Domain: Reasoning with Equations and Inequalities	**A-REI**
Clusters	• Solve systems of equations.

Conceptual Category: **Functions**	
Domain: Interpreting Functions	**F-IF**
Clusters	• Understand the concept of a function and use function notation
	• Interpret functions that arise in applications in terms of context.
	• Analyze functions using different representations. *Logarithmic and trigonometric functions*

Conceptual Category: **Functions**	
Domain: Building Functions	**F-BF**
Clusters	• Build a function that models a relationship between two quantities.
	• Build new functions from existing functions.

Conceptual Category: **Functions**	
Domain: Trigonometric Functions	**F-TF**
Clusters	• Extend the domain of trigonometric functions using the unit circle.
	• Model periodic phenomena with trigonometric functions.
	• Prove and apply trigonometric identities.

Conceptual Category: Geometry	
Domain: Similarity, Right Triangles, and Trigonometry	**G-SRT**
Clusters	• Apply trigonometry to general triangles

Conceptual Category: Geometry	
Domain: Expressing Geometric Properties with Equations	**G-GPE**
Clusters	• Translate between the geometric description and the equation for a conic section

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit #1: Functions

Write a description of the commonalities and other relationship between the given trigonometric function and the related circular function.

- Use algebraic, power, exponential, and transcendental functions to model real-world phenomena.
- Understand the interrelationships between the algebraic and transcendental functions.
- Explore graphs in three-dimensions.
- Explore functions with several variables.
- Explore the relationships between trigonometric, algebraic, polar, exponential, and logarithmic functions.
- Sketch the graph of trigonometric, polar, exponential, and logarithmic functions, with and without technology.
- Explore recursive functions with and without technology.
- Estimate the shapes of graphs of various functions.
- Describe the graphic relationship between a function and its inverse.
- Develop graphical techniques of solutions for problem situations involving functions.
- Graph linear and higher-order functions with and without technology.
- Explore proofs by mathematical induction.

<u>Functions – Interpreting Functions</u>

Interpret functions that arise in applications in terms of the context
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
 a. Focus on linear and exponential functions.
 b. Focus on linear, quadratic, and exponential functions.
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

Unit #1: Functions CONTINUED

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

- **c.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **d.** Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior.
- **e.** Graph simple exponential functions, indicating intercepts and end behavior.
- **f.** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

<u>Functions – Building Functions</u>

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from context.
  - **i.** Focus on linear and exponential functions.
  - **ii.** Focus on situations that exhibit quadratic or exponential relationships.
- **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

(+) **c.** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- **a.** Focus on transformations of graphs of quadratic functions, except for $f(kx)$.

F.BF.4 Find inverse functions.

- **a.** Informally determine the input of a function when the output is known.

(+) **b.** Read values of an inverse function from a graph or a table, given that the function has an inverse.

(+) **c**. Verify by composition that one function is the inverse of another.

Unit #1: Functions CONTINUED

(+) **d.** Find the inverse of a function algebraically, given that the function has an inverse.
(+) **e.** Produce an invertible function from a non-invertible function by restricting the domain.
(+) **F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Unit #2: Trigonometry

- Solve trigonometric identities graphically and analytically.
- Explore the relationships between trigonometric, algebraic, polar, exponential, and logarithmic functions.
- Sketch the graph of trigonometric, polar, exponential, and logarithmic functions, with and without technology.

Functions – Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

(+) **F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

(+) **F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

(+) **F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

(+) **F.T.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities.

(+) **F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Geometry – Similarity, Right Triangles and Trigonometry

Apply trigonometry to general triangles.

(+) **G.SRT.9** Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

(+) **G.SRT.10** Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.

(+) **G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.

Unit #3: Algebra, Number Systems and Vectors

Students demonstrate number sense including an understanding of number systems and of operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper-and-pencil, technology supported, and mental models.

- Estimate answers, demonstrate an understanding of the complex number system by developing facility with its operation.
- Compare and contrast the following number systems: real, rational, irrational, and the complex.
- Perform all arithmetic operations in the real, rational, irrational, and the complex number systems.
- Determine if given complex numbers are zeros of a given function.
- Determine, describe and use the inverse relationship between functions including exponential and logarithmic functions.
- Use mental computation and estimation to determine the reasonableness of answers when using technology.
- Construct and use matrices to describe and apply transformations.
- Use vectors to represent two pairs of directions and magnitudes and their sum or difference.
- Demonstrate an understanding of sequences, series, and other types of appropriate functions by determining convergences, divergences, and by finding limits.

Algebra – Seeing Structure in Expressions

Interpret the structure of expressions.

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Unit #3: Algebra, Number Systems and Vectors CONTINUED

Algebra – Arithmetic with Polynomials and Rational Expressions

Rewrite rational expressions.

A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

(+) **A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Algebra – Reasoning with Equations and Inequalities

Solve systems of equations.

(+) **A.REI.8** Represent a system of linear equations as a single matrix equation in a vector variable.

(+) **A.REI.9** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Number and Quantity – The Complex Number System

Perform arithmetic operations with complex numbers.

(+) **N.CN.3** Find the conjugate of a complex number; use conjugates to find magnitudes and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

(+) **N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

(+) **N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + ii\sqrt{3})^3 = 8$ *because* $(-1 + ii\sqrt{3})$ *has magnitude 2 and argument 120°.*

(+) **N.CN.6** Calculate the distance between numbers in the complex plane as the magnitude of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Unit #3: Algebra, Number Systems and Vectors CONTINUED

<u>Number and Quantity – Vector and Matrix Quantities</u>

Represent and model with vector quantities.

(+) **N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes, e.g., $\boldsymbol{vv}$, $|\ \boldsymbol{vv}\ |$, $||\boldsymbol{vv}||$, $\vec{vv}$.

(+) **N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

(+) **N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

(+) **N.VM.4** Add and subtract vectors.

- **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- **c.** Understand vector subtraction $\boldsymbol{vv} - \boldsymbol{ww}$ as $\boldsymbol{vv} + (-\boldsymbol{ww})$, where $-\ \boldsymbol{ww}$ is the additive inverse of $\boldsymbol{ww}$, with the same magnitude as $\boldsymbol{ww}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

(+) **N.VM.5** Multiply a vector by a scalar.

- **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as cc $(vvx, vvy) = (ccccx, ccccy)$.
- **b.** Compute the magnitude of a scalar multiple $cc\boldsymbol{vv}$ using $||cc\boldsymbol{vv}\ || = |cc|\ \boldsymbol{vv}$. Compute the direction of $cc\boldsymbol{vv}$ knowing that when $|\ cc\ |\ \boldsymbol{vv} \neq 0$, the direction of $cc\boldsymbol{vv}$ is either along $\boldsymbol{vv}$ (for $cc > 0$) or against $\boldsymbol{vv}$ (for $cc < 0$).

Perform operations on matrices and use matrices in applications.

(+) **N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

(+) **N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

(+) **N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.

(+) **N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

Unit #3: Algebra, Number Systems and Vectors CONTINUED

(+) **N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication analogous to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
(+) **N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
(+) **N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Unit #4: Analytic Geometry

- Explore general relationships between the algebraic description and the graphs of the special properties of each conic section with and without technology.
- Graph using polar coordinates.
- Explore the relationships between trigonometric, algebraic, polar, exponential, and logarithmic functions.
- Sketch the graph of trigonometric, polar, exponential, and logarithmic functions, with and without technology.
- Demonstrate an understanding of polar and parametric equations by relating the equations to the graphs with and without technology.
- Expand and extend the idea of vectors and linear algebra to higher dimensional situations.
- Develop and communicate arguments about limit situations.

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.
(+) **G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.